

not affect the marginal rates of substitution between different factors of production for given factor ratios.

These substantive conclusions derive from two conclusions of methodological interest:

1. The appropriate model at the firm level is a statistical cost function which includes factor prices and which is uniquely related to the underlying production function.

2. At the firm level it is appropriate to assume a production function that allows substitution among factors of production. When a statistical cost function based on a generalized Cobb-Douglas production function is fitted to cross-section data on individual firms, there is evidence of such substitution possibilities.

Inadequacies in the estimation of capital costs and prices and in the treatment of transmission suggest, however, that a less aggregative approach is called for. On a less aggregative level, it may be possible to produce more adequate measures of capital and to introduce transmission explicitly. A simple model of optimal behavior on the part of the firm may then allow us to combine this information in a way that will yield more meaningful results on returns to scale at the firm level.

APPENDIX A

A Relation Between Returns to Scale at the Plant Level and at the Firm Level for an Electric Utility

Consider a firm that produces x_i units in each of n identical plants. If plants and demand are uniformly distributed, all plants will produce identical outputs, so that the total output produced will be nx , where x is the common value. Under these circumstances, a general formula that has been developed by electrical engineers to express transmission losses [8] reduces to

$$(A.1) \quad y = bn^2x^2,$$

where y is the aggregate loss of power. That is, with uniformly distributed demand and identical plants, transmission losses are proportional to the square of total output.

If z is delivered power, we have

$$(A.2) \quad z = nx - y = nx - bn^2x^2.$$

Let $c(x)$ be the cost of producing x units in one plant. Production costs of the nx units are thus $nc(x)$. And let $t = T(n, x)$ be the cost of maintaining a network with n plants, each of which produces x units. We may expect that

t increases with x , $\partial T/\partial x > 0$, since larger outputs require more and heavier wires and more and larger transformers. However, t may or may not increase with n . It is likely to decrease with n if the expense of operating and maintaining long transmission lines is large relative to the cost of a number of short lines, and likely to increase if the converse is true.

The total cost of delivering an amount z of power $\Gamma(z)$ is the sum of production costs of a larger amount of power and transmission costs:

$$(A.3) \quad \Gamma(z) = nc(x) + T(n, x).$$

Suppose that the firm chooses the number and size of its plants in order to minimize $\Gamma(z)$ for any given z . The values of n and x that minimize $\Gamma(z)$ subject to (A.2) are given by solving

$$(A.4) \quad c(x) + \frac{\partial T}{\partial n} - x\lambda\mu = 0,$$

$$(A.5) \quad nc'(x) + \frac{\partial T}{\partial x} - n\lambda\mu = 0,$$

$$(A.6) \quad z - (nx - bn^2x^2) = 0,$$

where

$$(A.7) \quad \begin{aligned} \mu &= 1 - 2bnx \\ &= \frac{z - y}{nx}. \end{aligned}$$

The degree of returns to scale at the plant level, $p(x)$, may be defined as the reciprocal of the elasticity of production costs with respect to output:

$$(A.8) \quad p(x) = \frac{c(x)}{xc'(x)}.$$

It follows from (A.4), (A.5), and (A.8) that

$$(A.9) \quad p(x) = 1 + \frac{t}{(nx)c'(x)}(e_x - e_n),$$

where

$$e_x = \frac{x}{t} \frac{\partial T}{\partial x}, \quad e_n = \frac{n}{t} \frac{\partial T}{\partial n}.$$

Since nx , t and $c'(x)$ are positive, it follows that returns to scale are greater or less than one, according to whether the elasticity of transmission costs with respect to output exceeds or falls short of the elasticity with respect to number of plants. If transmission costs decrease with a larger number of plants, then under the particular assumptions made here, the firm will

operate plants in the region of increasing returns to scale. It may nonetheless operate as a whole in the region of decreasing returns to scale.

Let $P(z)$ be the degree of returns to scale for the firm as a whole when it delivers a supply of z units to its customers:

$$(A.10) \quad P(z) = \frac{\Gamma(z)}{z\Gamma'(z)}.$$

It is well known that the Lagrangian multiplier λ is equal to marginal cost; hence, from (A.5),

$$(A.11) \quad \Gamma'(z) = \lambda = \frac{1}{n\mu} \left[nc'(x) + \frac{\partial T}{\partial x} \right].$$

Substituting for $\Gamma'(z)$ from (A.11), μ from (A.7), and $\Gamma(z)$ from (A.3), we obtain the following expression for $P(z)$:

$$(A.12) \quad \begin{aligned} P(z) &= \frac{\Gamma(z)}{z} \cdot \frac{n(z-y)}{nx[nc'(x) + \partial T/\partial x]} \\ &= \left(1 - \frac{y}{z}\right) \frac{nc(x) + t}{n[xc'(x)] + x(\partial T/\partial x)}. \end{aligned}$$

By definition,

$$p(x) = \frac{c(x)}{xc'(x)},$$

hence

$$(A.13) \quad P(z) = p(x) \left(1 - \frac{y}{z}\right) \frac{nc(x) + t}{nc(x) + [p(x)e_x]t}.$$

Neglecting the last term in the product on the right-hand side of (A.13) for the moment, we see that returns to scale at the firm level will typically be less than at the plant level, solely because of transmission losses; how much less depends on the ratio of losses to the quantity of power actually delivered. The final term in the product is a more complicated matter: If there are increasing returns to scale and if the costs of transmission increase rapidly with the average load (i.e., $e_x > 1$), then it is clear that the tendency toward diminishing returns at the level of the individual firm will be reinforced. It is perfectly possible under these circumstances that firms will operate individual plants in the range of increasing returns to scale and yet, considered as a unit, be well within the range of decreasing returns to scale.

Although this argument rests on a number of extreme simplifying assumptions, it nonetheless may provide an explanation for the divergent views and findings concerning the nature of returns to scale in electricity supply. Davidson [3] and Houthakker [9], for example, hold that there are diminishing returns to scale, while much of the empirical evidence and

many other writers support the contrary view. The existing empirical evidence, however, refers to individual plants, not firms, and many writers in the public-utility field may have plants rather than firms in mind.

APPENDIX B

The Data Used in the Statistical Analyses

Estimation of equation (7) from cross-section data on individual firms in the electric power industry requires that we obtain data on production costs, total physical output, and the prices paid for fuel, capital, and labor. Data on various categories of cost are relatively easy to come by, although there are difficulties in deriving an appropriate measure of capital costs. Price data are more difficult to come by, in general, and conceptual as well as practical difficulties are involved in formulating an appropriate measure of the "price" of capital. Such problems are, in fact, the *raisons d'être* for Model B, which permits us to ignore capital prices altogether.

A cross section of 145 firms in 44 states in the year 1955 was used in the analyses. The firms used in the analysis are listed in Appendix C. Selection of firms was made primarily on the basis of data availability. The various series used in the analyses were derived as follows.

B.1. Production Costs

Data on expenditures for labor and fuel used in steam plants for electric power generation are available by firm in [6], but the capital costs of production had to be estimated. This was done by taking interest and depreciation charges on the firm's entire production plant and multiplying by the ratio of the value of steam plant to total plant as carried on the firm's books. Among the shortcomings of this approach, three are worthy of special note:

(a) For many well-known reasons, depreciation and interest charges do not reflect capital costs as defined in some economically meaningful way. Furthermore, depreciation practices vary from firm to firm (there are about four basic methods in use by electric utilities), and such variation introduces a noncomparability of unknown extent.

(b) The method of allocation used to derive our series assumes that steam and hydraulic plants depreciate at the same rate, which is clearly not the case.

(c) Because of their dependence on past prices of utility plant, the use of depreciation and interest charges raises serious questions about the relevant measure of the price of capital. The use of a current figure is clearly inappropriate, but unless we are prepared to introduce the same magnitude on both

sides of the equation, it is difficult to see how else the problem can be handled.

B.2. Output

Total output produced by steam plant in kilowatt hours during the entire year 1955 may be obtained from [6]. This was the series used, despite the fact that the peak load aspect of output is thereby neglected. Since the distribution of output among residential, commercial, and industrial users varies from firm to firm, characteristics of the peak will also vary and this in turn will affect our estimate of returns to scale if correlated with the level of output.

B.3. Wage Rates

At the time this study was undertaken, I was unaware of the existence of data on payroll and employment by plant contained in [5]; hence, inferior information was used to obtain this series. Average hourly earnings of utility workers (including gas and transportation) were available for 19 states from Bureau of Labor Statistics files. A mail survey was made of the State Unemployment Compensation Commissions in the remaining 29 states. All replied, but only ten were able to supply data. A regression of the average hourly earnings of utility workers on those for all manufacturing was used to estimate the former for states for which it was unavailable. The resulting state figures were then associated with utilities having the bulk of their operations in each state. In only one case, Northern States Power, were operations so evenly divided among several states that the procedure could not be applied. In this case an average of the Minnesota and Wisconsin rates was employed.

B.4. Price of Capital

As indicated, many practical and conceptual difficulties were associated with this series. Be that as it may, what was done was as follows: First, an estimate of the current long-term rate at which the firm could borrow was obtained by taking the current yield on the firm's most recently issued long-term bonds (obtained from Moody's Investment Manual). These were mainly 30-year obligations, and in all cases had 20 or more years to maturity. This rate was in turn multiplied by the Handy-Whitman Index of Electric Utility Construction Costs for the region in which the firm had the bulk of its operations [4, p. 69]. Two shortcomings worth special mention are:

- (a) The neglect of the possibility of equity financing by the method.
- (b) The fact that the Handy-Whitman Index includes the construction costs of hydraulic installations.